2. A. I. Leont'ev and A. G. Kirdyashkin, "Heat transfer under free convection in horizontal slots in a large volume above a horizontal surface," Inzh.-fiz. Zh., 9, No. 1 (1965).
3. A. G. Kirdyashkin, "Friction and heat transfer under thermal gravitational convection in a centrifugal force field," Author's Abstract of Candidates Dissertation, ITF Sib. Otd. Akad. Nauk SSSR, Novosibirsk (1966).
4. H. Schlichting, Boundary Layer Theory, McGraw-Hill (1968).
5. A. P. Burdukov and G. G. Kuvshinov, "Investigation of the boiling mechanism by the electrodiffusion method," in: Enhancement of Heat Transfer in Power Equipment [in Russian], Izd. ITF Sib. Otd. Akad. Nauk SSSR, Novosibirsk (1977).
6. A. P. Burdukov, G. G. Kuvshinov and V. E. Nakoryakov, "Special features of heat transfer in bubble boiling of an underheated liquid in a large volume at subatmospheric pressures," in: Heat Transfer with Boiling and Condensation [in Russian], Izd. ITF Sib. Otd. Akad Nauk SSSR, Novosibirsk (1978).

OPTIMIZATION OF THE COMPRESSION OF A SPHERICAL MASS OF GAS

V. I. Kosarev and A. M. Svalov

UDC 533.6.011

One of the basic problems in the study of combustion processes is that of achieving ignition conditions — high enough temperature and density of the combustible medium so that the combustion reaction, once having been started, can effectively continue. To satisfy the ignition conditions it is necessary to impose conditions on the characteristics of the energy source which initiates the reaction. The description of processes taking place during combustion requires taking account of various physical factors such as heat conduction, degeneracy effects, radiation, etc., and the fact that the material being compressed may be a two-component medium — a plasma. Therefore, an analytic study of the problem in the whole volume is practically impossible, and the problem must be separated into parts, each of which can be described by a simpler mathematical model, with a subsequent numerical verification of the limits of admissibility of the simplifying assumptions. Thus, the compression of a material is satisfactorily described by the action of a piston on an ideal compressible fluid [1-7]. The compression efficiency is commonly characterized by the quantity $\langle\rho R\rangle = \int_0^r \rho dr$, where ρ is the density of the medium being compressed. It is expedient to divide the solutions under study into two groups; the first group consists of the solutions in which $\langle\rho R\rangle$ reaches the required values when constant input data are used [1-3], and the second group includes all the remaining solutions. It is natural to require simple initial distributions of the hydrodynamic functions so that the proposed schemes can be realized practically by means available. However, taking account of the advance of technical feasibilities and certain advantages of solutions of the second group, it is necessary to study all schemes which ensure the achievement of large values of $\langle\rho R\rangle$. Cvalov [7] showed that for special distributions of the initial values solutions exist for which $\langle\rho R\rangle$ becomes infinite at a certain instant for a finite mass and a finite expenditure of energy in the compression, whereas the use of a compression scheme with a uniform deformation [4-6] leads to a linear dependence of $\langle\rho R\rangle$ on the added energy for $\gamma = 5/3$. The solutions given in [7] are self-similar near the origin, and satisfy a system of ordinary differential equations [8]. This system depends on two parameters $\varkappa$ and δ, and the unknown functions can be written in the form

$$u = \frac{r}{t} V(\lambda), \quad \rho = \frac{a}{r^{k+3} t^{s}} R(\lambda), \quad p = \frac{a}{r^{k+1} t^{s+2}} P(\lambda),$$

$$\lambda = r/(bt^{\delta}),\ z(\lambda) = \gamma P(\lambda)/R(\lambda),\ \varkappa = (s + 2 + \delta\ (k+1))/\gamma,$$

where u, ρ, and p are respectively the velocity, density, and pressure of a particle, r and t are space and time coordinates, and a, b, k, s, $\varkappa$, and δ are arbitrary constants.

Moscow. Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, No. 4, pp. 54-59, July-August, 1983. Original article submitted July 7, 1982.

 0021-8944/83/2404-0494$07.50

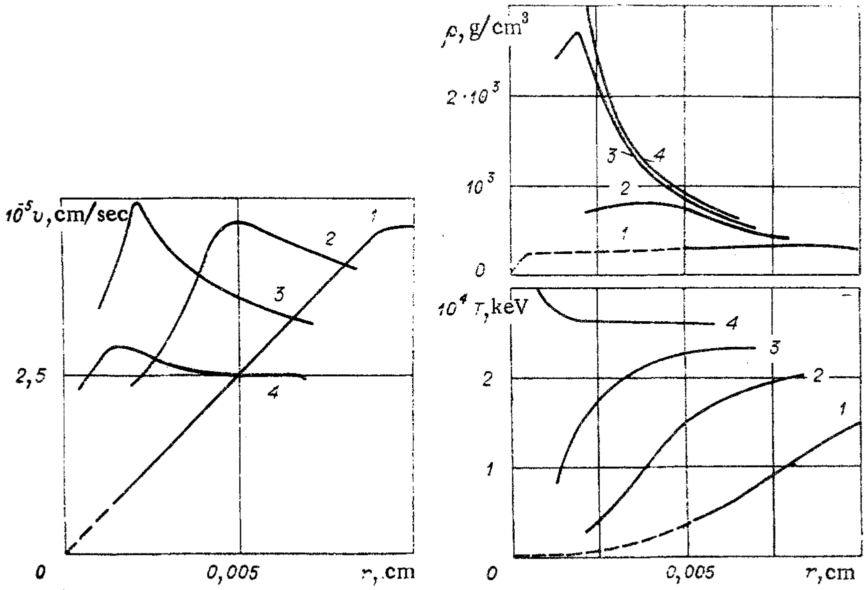

Fig. 1

The values of the parameters ϰ and δ for which there are self-similar solutions having the above property were determined in [7]. The solutions of this class ensure arbitrarily large values of the compression efficiency index <ρR> in the compression of a finite mass, and a finite expenditure of energy, and are optimal with respect to energy expenditure as compared with other types of solutions leading to superhigh compression.

The question naturally arises of how taking account of the accompanying physical phenomena such as heat conduction, etc., affects the character of the compression. Since it is difficult to take account of these factors analytically, we performed numerical studies of compression schemes based on the solutions mentioned. Another reason for making numerical calculations was to find how the characteristics of the initial distribution and the law of motion of the compressing piston influence the compression efficiency.

In the final analysis we started from the fact that numerical experiments permit an estimate of the practical significance of self-similar compression, since if deviations from it either as a result of taking account of additional physical factors or because of violation of the required initial distributions leads to a considerable decrease in the compression efficiency, this will indicate the exceptional character of self-similar compression, its weak stability, and the difficulty of its realization.

We chose a solution describing the compression of a dense target for numerical study. This solution is represented by an integral curve in the plane of the self-similar variables (z, V) which passes through the two singular points described in [7].

The numerical solution of the problem is reduced to the following:

the integration of ordinary differential equations which are satisfied by the self-similar solution in order to calculate the initial distributions of gasdynamic functions. In the neighborhood of the singular points the asymptotic formulas in [7] are used to obtain the solution;

the calculation of the trajectory and the conditions at the piston which close the problem for the gasdynamic equations in Lagrangian variables;

the numerical solution of the last problem, i.e. the integration of the system of partial differential equations.

It is known that a self-similar solution determines a certain class consisting of an infinite set of solutions. Calculations were performed for the following specific scales of the compression process studied:

(A) $\begin{cases} R_0 = 10^{-2} \text{ cm} — \text{initial radius of compressible target,} \\ \tau = 10^{-8} \text{ sec} — \text{time during which collapse is reached in the self-similar solution} \end{cases}$

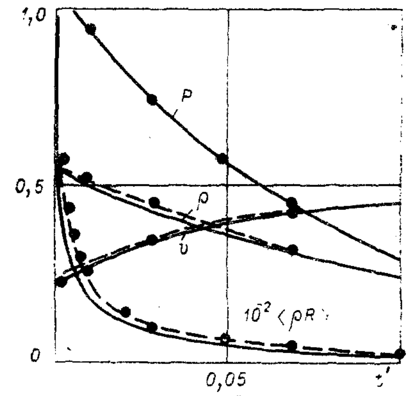

Fig. 2

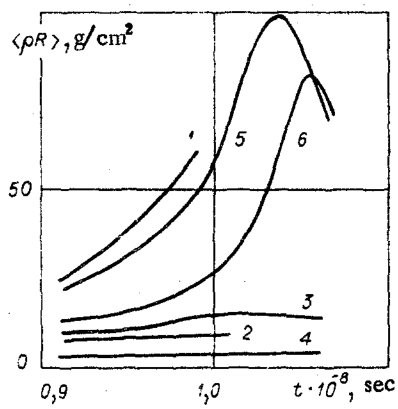

Fig. 3

(the characteristic scales of the other gasdynamic parameters were as follows: density $\rho \sim 10^2$ g/cm^3, temperature $T \sim 10^{-4}$ keV, speed $u \sim 10^5$ cm/sec);

(B) $R_0 = 10^{-2}$ cm, $\tau = 10^{-10}$ sec ($\rho \sim 1$ g/cm^3, $T \sim 1$ keV, $u \sim 10^7$ cm/sec).

We used the ideal gas model with values of the physical constants for a DT sphere.

We present selected results of the calculations performed. Curves 1-4 of Fig. 1 show the distributions of gasdynamic parameters at times $t = 0$, 0.52×10^{-8}, 0.81×10^{-8}, and 0.99×10^{-8} sec respectively. For the same orders or magnitude the computed values for cases (A) and (B) become indistinguishable, and are plotted as single curves.

This clearly shows that the calculated solution is self-similar at each instant and at each point. The calculations showed, however, that the singularities of the initial distributions near the center of the compressible sphere are most important in maintaining the self-similar character of the compression shown by the dashed parts of the curves 1 in Fig. 1. Of the 60 nodes at which calculations were made, 30 were near the center. At these nodes the singularities were taken into account by calculating the initial values from the asymptotic formulas of [7].

Figure 2 illustrates the conditions at the "piston" — the outer boundary of the region calculated. Here t' is the "self-similar" time. The graphs are plotted in relative units. Figure 2 also shows the resulting dependence of $\langle\rho R\rangle$ for the self-similar regime. The points mark the results of a numerical solution of the gasdynamic equations. Of course infinite compression is not reached in the calculated data. The maximum value of the ratio of $\langle\rho R\rangle$ to its initial value ≈ 60. Nevertheless, the numerical and self-similar solutions clearly agree, both with respect to the integral characteristic $\langle\rho R\rangle$ and with respect to the parameters at the piston.

One of the basic results of the calculations performed is the verification of the effect of heat conduction on the compression regimes studied. Heat transfer was taken into account by using the model in [9], taking the thermal conductivity as a power function of the temperature.

It was found that the distributions of the gasdynamic parameters remain as before (Fig. 1) even in case (B) when the characteristic values of the temperature are rather high; i.e. the self-similar regime studied is determined mainly by the hydrodynamics over a rather wide range of scales of the process: (A), (B). In principle this conclusion was to be expected. On the basis of the scales shown it is easy to obtain a rough estimate of the significance of the separate terms in the equations describing the phenomena in question. The conclusion stated follows from this estimate. The numerical results confirm its validity for a strong nonlinearity of the actual heat transfer process.

The following results relate to a study of the effect on the compression characteristics of the initial distributions and conditions at the piston which differ from self-similar.

Figure 3 shows the time dependence of $\langle\rho R\rangle$ for various initial conditions: ρ_k^0, T_k^0, v_k^0, where the subscript k indicates a distribution of initial values over the cells of the difference net. The values of the pressure P_B on the piston, which serves as a boundary condition for the solution of the gasdynamic equations, were varied. The curves in Fig. 3

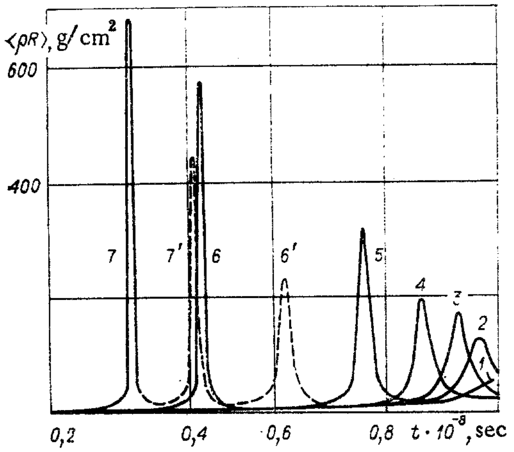

Fig. 4

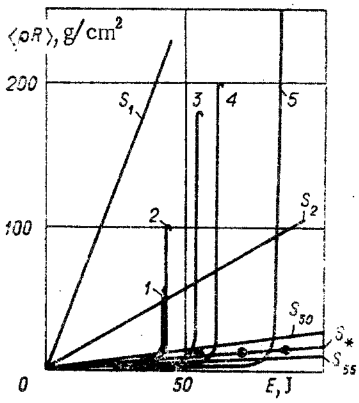

Fig. 5

correspond to the following values of the parameters: 1) initial values and pressure on the piston correspond to self-similar compression; 2) $T_k^0 = 10^{-4}$ keV, $\rho_k^0 = 250$ g/cm³, v_k^0 and P_B correspond to self-similar compression; 3) $T_k^0 = 10^{-4}$ keV, $\rho_k^0 = 250$ g/cm³, v_k^0 from the self-similar solution, $P_B = P_* = 5.37 \times 10^9$ Pa; 4) T_k^0 and ρ_k^0 from the self-similar solution, $v_k^0 = 0$, $P_B = P_*$; 5) $T_k^0 = 10^{-4}$ keV for $k \geqslant 25$, for $k < 25$ the self-similar distribution, $\rho_k^0 = 250$ g/cm³, v_k^0 corresponds to the self-similar regime, $P_B = P_*$; 6) differs from 5 only in the initial temperature distribution: $T_k^0 = 10^{-5}$ keV for $k < 25$, 10^{-4} keV from $k \geqslant 25$. The value $P_* = 5.37 \times 10^9$ Pa used here as the limiting pressure was chosen so that the work performed by the piston in this case, i.e. the energy added from the outside, is equal to the corresponding work in self-similar compression. The results show clearly the important effect of the particular character of the initial distributions near the center of the compressible sphere on the compression efficiency. On the other hand, it is clear that in retaining the self-similar initial velocity distribution, the modeling of initial values of the temperature by a simple power function for a uniformly distributed initial density leads to a high compression efficiency, which is of definite interest in view of the above remark on the possibility of the practical realization of these or other compression schemes. Figure 4 shows the time dependence of $\langle\rho R\rangle$ for various values of the pressure on the piston: 1) self-similar compression; P_B = 6.63×10^9, 8.0×10^9, 10.0×10^9, 15.0×10^9, 50.0×10^9, 100.0×10^9 Pa for curves 2-7 respectively; 6' and 7' are the continuations of curves 6 and 7 for larger values of t. The initial values for all cases are distributed according to the self-similar law. It is characteristic that at a certain instant the degree of compression reaches a very high maximum in all the regimes shown. The maximum of $\langle\rho R\rangle$ increases markedly with an increase in external pressure, which is obviously related to the increase in the energy added to the target as a result of external compressive forces. For $P_B = 15$, 50, and 100×10^9 Pa, a second maximum occurs as a result of shock compression in these cases.

The results obtained enable us to draw certain conclusions about the influence of the added energy on the compression efficiency in the regimes studied. Figure 5 shows the dependence of these values of $\langle\rho R\rangle$ on the energy added to the target: 1) self-similar compression; $P_B = 5.37 \times 10^8$, 8.0×10^9, 10.0×10^9, 15.0×10^9 Pa for curves 2-5 respectively. A characteristic feature of the self-similar regime investigated is the presence of a sharp peak in the distribution of $\langle\rho R\rangle$ at a certain value of the energy (theoretically the value of $\langle\rho R\rangle$ should become infinite at this point). This characteristic is preserved also for a constant pressure on the piston and for self-similar initial distributions of the functions; the only changes are in the maximum value of $\langle\langle\rho R\rangle$ and the point at which it is reached.

For certain exact solutions of the gasdynamic equations the function $\langle\rho R\rangle$ (E) can be expressed in explicit form. For example, for solutions with uniform information used in modeling compression phenomena [4-6], this function is linear. For the case studied in [6] it has the form ($\gamma = 5/3$)

$$\langle\rho R\rangle = \frac{1}{4(18\pi)^{1/3}} \frac{E}{SM^{2/3}}. \quad (1)$$

The linear form of the function $\langle\rho R\rangle$ (E) is preserved also when the compression process is described approximately by the formulas of uniform adiabatic flow:

$$\langle\rho R\rangle=\left(\frac{2}{9\pi}\right)^{1/3}\frac{E}{SM^{2/3}}. \quad (2)$$

Here E, M, and S are respectively the energy, mass, and entropy function ($S = P/\rho^{\gamma}$) of the target.

As one might have expected, the degree of compression is characterized by the entropy of the system, and in the limit the compression may be indefinitely high for a finite energy of the system if S = 0. A characteristic of the solutions studied in the present paper is the fact that the function S actually vanishes, but only at the center of the target. It remained finite when averaged over the mass. The straight lines in Fig. 5 represent Eq. (2) for various values of S (the subscripts indicate the number of the computational cell which corresponds to this value); the points mark the straight line corresponding to the value of S, averaged over the mass, for the solution studied. It is clear from Eq. (1) that the coefficient of E for compression with a uniform deformation is about 6.35 times smaller than the coefficient in (2). Thus, if one admits the possibility of generating arbitrary input data, the self-similar solutions investigated describe a compression scheme leading to the required value of the compression efficiency which is more economical with respect to energy expenditure than those in [4-6].

The nearly self-similar solutions studied in the present paper with initial distributions simpler than self-similar and with constant pressure on the compressing piston also have this property.

LITERATURE CITED

1. I. B. Zababakhin and V. A. Simonenko, "The spherical centered compression wave," Prikl. Mat. Mekh., 42, 573 (1978).
2. Ya. M. Kazhdan, "Adiabatic compression of a gas by means of a spherical driver," Prikl. Mekh. Tekh. Fiz., No. 1, 23 (1977).
3. S. I. Anisimov, M. F. Ivanov, and N. A. Inogamov, "Dynamics of laser compression and heating of simple targets," Preprint Inst. Teor. Fiz., Akad. Nauk SSSR (1977).
4. N. V. Zmitrenko and S. P. Kurdyumov, "Self-similar cases of compression of a finite plasma mass," Dokl. Akad. Nauk. SSSR, 218, No. 6 (1974).
5. R. E. Kidder, "Theory of homogeneous isentropic compression and its application to laser fusion," Nucl. Fusion, 14, 53 (1974).
6. V. V. Demchenko, "Comparative study of certain compression processes," Zh. Vychisl. Mat. Mat. Fiz., 19, No. 2 (1979).
7. A. M. Svalov, "Compression of spherical targets," Izv. Akad. Nauk SSSR, Mekh. Zhidk. Gaza, No. 3 (1982).
8. L. I. Sedov, Similarity and Dimensional Methods in Mechanics, Academic Press, New York (1959).
9. L. Spitzer, Physics of Fully Ionized Gases, Interscience, New York (1956).